Appendix A

Further Details of Vector Autoregression

Vector Autoregression (VAR) is a technique for analyzing time series data that is utilized throughout the book. Since it is a technique that is not broadly taught in the social sciences, I provide a description of the method here that can be referenced whenever the technique is applied. A more detailed but still accessible introduction to the method and its application in political science can be found in Freeman, Williams & Lin (1989).

I use a variety of analytical techniques, but much of the analysis assesses changes over time in economic inequality, other economic conditions, and politics. The core questions of the book generally revolve around some variant of the following: how do changes in inequality affect politics and how do changes in politics affect inequality? Those questions are inherently dynamic in nature—they imply a process that unfolds over time.

Many variants of time series analysis are available to answer questions about movement over time in key variables, and most of these techniques are rooted in a classic regression framework in which a dependent variable Y is modeled as a function of key explanatory variables and control variables that are "held-constant" statistically. The idea is to isolate the causal effect of the key explanatory variables by examining co-variation over time and observing temporal ordering. One typical form of time series regression could look something like this:

$$Y_t = \alpha_0 + \beta_0 Y_{t-1} + \beta_1 X_t + \beta_2 Z_t + \epsilon_t \tag{A.1}$$

The subscripts here make reference to points in time, so t means the current period and $t-1$ means the previous period. X_t is an explanatory variable in the current period while Z_t is a control variable in the current period. Y_{t-1} refers to the lagged value of the dependent variable, and explains why a model such as this is often referred to as a lagged dependent variable (LDV) model. This model, then, attempts to estimate the effect of X on Y while statistically controlling for previous values of Y along with values of Z.

The LDV model is a useful foundation for understanding VAR models. The key distinguishing characteristic of an LDV model is the presence of a lag of the dependent variable on the right hand side of the equation. This modeling strategy has a number of benefits. First, one of the core challenges of time series analysis in terms of statistical inference is overcoming the problem of autocorrelation, or the tendency of current values of a series to be correlated with previous values. Autocorrelation can wreak havoc on our ability to draw inferences from observed time series data. Estimating an LDV model often ameliorates concerns about autocorrelation by explicitly including past values of the dependent variable in the model. Previous values of other explanatory variables are also implicitly included in an LDV model to the extent that previous values of the Y are explained by previous values of X and Z.

Second, inclusion of lagged Y as an explanatory variable provides for the possibility of *dynamic causation*, or effects that are spread out over time. Consider equation A.1 above. β_1 provides an estimate of the contemporaneous effect of X on Y. But if there is an effect of X on Y, there is also an effect of X_{t-1} on Y_{t-1}. The effect of X_{t-1} on current values of Y flows indirectly via the effect of Y_{t-1}. You can work your way back in time infinitely with this logic. All of this implies that the overall effect of X on Y combines β_1 with the accumulated historical effect via β_0. That is, the effect of X on Y does not happen all at once but is spread out over multiple time periods. While it can certainly be the case that effects in time series data are static, occurring all at one point in time (and the model can be modified to account for this if need be), it is quite common for causation to be dynamic when dealing with time series data.

Third, LDV models provide substantial protections against problems generated by incorrectly excluding explanatory variables. As is well known, when some variable Z that explains both X and Y is excluded from a regression analysis, spurious results can appear in which the effect of X is inaccurately estimated. Perfectly specified models don't likely exist, so some degree of model misspecification is almost always present in regression analysis. Controlling for lagged Y in a time series analysis provides a shorthand way to control for a host of un-modeled variables. The lag of any excluded variable Z that affects the current value of Y will by definition affect the lag of Y. So by controlling for the lag of Y, lagged values of unmodeled potential confounders are implicitly modeled. To the extent that the current value of a variable is not affected by previous values, of course, the protections against model misspecification are minimized. But given the typical prevalence of autocorrelation in time series data, LDV models are often very useful guards against the exclusion of explanatory variables. In fact, one of the most commonly identified weaknesses of LDV models is that they can *underestimate* the effects of the explanatory variables of interest in the model (Keele & Kelly 2006).

The LDV model and its derivatives are the most commonly estimated time series model in applied applications. This family of models works fairly effectively in many situations, but it is by no means always appropriate. The questions I seek to answer in this book cannot be answered with this type of model. The problem is that I am interested in the interplay of variables over time, in particular how politics and economic inequality affect each other. The model described above has theoretically defined outcomes and explanations. One variable is what we're trying to explain and the other variables are the potential explanation. But the idea of an inequality trap implies feedback. Outcomes in one part of the analysis are explanations in other parts. And to the extent that the feedback implied by an inequality trap is indeed present, estimating a simple LDV model will fail at generating valid inferences.

We need a model that can cope with a theoretical framework that anticipates causation flowing in multiple directions—a model that does not simply assume that the relationship between politics and economic inequality only goes only one way. The VAR model does just that (Box-Steffensmeier, Freeman, Hitt & Pevehouse 2014). And I started with a description of the LDV model above because VARs are related to LDV models in a variety of ways.

In a VAR, the variables in the model are part of a "system" in which each variable is potentially both a cause and an effect. Instead of estimating a single equation in which there is an outcome and the left side and several explanations on the right side, a VAR estimates multiple equations that allow for the possibility of contemporaneous and lagged feedback between the variables in the system. A three variable VAR can be expressed this way:

$$\begin{bmatrix} Y_{1,t} \\ Y_{2,t} \\ Y_{3,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \beta^1_{1,1}, \beta^1_{1,2}, \beta^1_{1,3} \\ \beta^1_{2,1}, \beta^1_{2,2}, \beta^1_{2,3} \\ \beta^1_{3,1}, \beta^1_{3,2}, \beta^1_{3,3} \end{bmatrix} \begin{bmatrix} Y_{1,t-1} \\ Y_{2,t-1} \\ Y_{3,t-1} \end{bmatrix} + \begin{bmatrix} \beta^p_{1,1}, \beta^p_{1,2}, \beta^p_{1,3} \\ \beta^p_{2,1}, \beta^p_{2,2}, \beta^p_{2,3} \\ \beta^p_{3,1}, \beta^p_{3,2}, \beta^p_{3,3} \end{bmatrix} \begin{bmatrix} Y_{1,t-p} \\ Y_{2,t-p} \\ Y_{3,t-p} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{bmatrix} \tag{A.2}$$

What equation A.2 shows is that for each variable in a VAR system, an equation is estimated in which lags of all variables in the system are included on the right hand side. In a three variable system, then, there are three equations that are estimated simultaneously. For each Y, p lagged values of Y and p lags of the other variables in the system are included. This equation shows that the VAR is indeed connected to the LDV model above. The key difference is that effects flow in multiple directions in a VAR model. Another important difference is that only values from previous time points are allowed to explain current values of any variable in the system. In a VAR system contemporaneous values of an explanatory variable and an outcome variable are not analyzed. Rather, a VAR estimates the effect of past values of explanatory variables while controlling for past values of outcome variables. This helps to bolster the ability to make causal inferences.

Still, VARs are rooted in an observational research design that is inherently correlational. And several important determinations must be made prior to the estimation of a VAR. First and foremost, VARs can only be estimated when the variables in the system are either stationary or co-integrated. For each of the VARs reported in this book, I begin by testing each variable in the system for a unit root using a combination of augmented Dickey-Fuller (Dickey & Fuller 1979) and KPSS (Kwiatkowski, Phillips, Schmidt & Shin 1992) tests. When data are non-stationary I test for cointegration prior to estimation and note the results of these tests. Second, the number of lags to include in the model can be a consequential decision. I utilize a test that seeks to maximize the fit of the model based on the Schwartz Bayesian Information Criterion, but at times I estimate models with varying lag lengths and mention whether this selection dramatically changes results.

One of the most complicated aspects of VAR analysis is interpreting results. Particularly as models include increasing numbers of variables and the number of lags increases, the number of coefficients estimated proliferates. This means that individual coefficients will be estimated imprecisely and that using individual coefficients to determine the size of effects is inappropriate. Additionally, since the most general versions of a VARs allow all of the variables in the system to affect each other, there are direct and indirect effects present. That is, while one variable (Y_1) might directly affect another variable in the system (Y_2), that same variable might have indirect effects that happen via effects on other variables in the system (Y_1 affecting Y_3 which then affects Y_2 for example).

Innovation accounting refers to a group of techniques to make inferences about the size, direction, and timing of effects based on VAR models. The technique I will rely on most heavily is Impulse Response Functions (IRFs). An impulse response function uses information from the VAR model to calculate the effect of some shift in one variable on another variable in the system. I typically report IRFs based on a one standard deviation shift (or impulse or shock). A simple IRF reports the effect of this shock at each point in time for a specified number of periods, starting at the onset of the shock. The IRF, then, provides information about how large and how long a shock in one variable has effects on another variable. I report orthoganalized cumulative IRFs which add up the effects over time to capture the total effect of a shock at a given point in time after the shock. So the cumulative effect of a shock ten periods after the onset of the shock would be the effect at lag 10 plus the effect at all lower lags.

IRFs, importantly, capture not just the direct effects of one variable on an outcome, but also the indirect effects that that variable has via other variables in the system. Constructing an IRF requires an assumption about the ordering of contemporaneous correlations, known as a Choleski decomposition. Whenever I report the results an IRF, you will see a note about the assumed causal ordering of the variables in the system and some theoretical justification for the assumption. I will also note whether changing the assumption about causal ordering affects the results.